TITLE: ROLLING IN THE MONEY!

Student/Class Goal
Students want to obtain the best deal when comparing prices.

Outcome (*lesson objective)*
Students will compare payment options expressed in a variety of units and representational forms. Students will apply the ECPC (Estimate-Convert-Plot-Compare) strategy for solving complex price comparison tasks.

Time Frame

2 hours

Standard *Use Math to Solve Problems and Communicate*

NRS EFL 5-6

Number Sense	Benchmarks	Geometry & Measurement	Benchmarks	Processes	Benchmarks
Words to numbers connection		Geometric figures		Word problems	5.25, 6.26
Calculation	5.1, 6.1	Coordinate system	**5.7, 6.7**	Problem solving strategies	**5.26, 6.27**
Order of operations		Perimeter/area/volume formulas		Solutions analysis	5.27, 6.28
Compare/order numbers	**5.3. 6.3**	Graphing two-dimensional figures		Calculator	
Estimation	**5.4, 6.4**	Measurement relationships		Math terminology/symbols	
Exponents/radical expressions		Pythagorean theorem		Logical progression	
Algebra & Patterns	**Benchmarks**	Measurement applications		Contextual situations	5.31, 6.32
Patterns/sequences	5.15, 6.15	Measurement conversions	5.13, 6.13	Mathematical material	
Equations/expressions		Rounding	5.14, 6.14	Logical terms	
Linear/nonlinear representations	5.17, 6.17	**Data Analysis & Probability**	**Benchmarks**	Accuracy/precision	
Graphing		Data interpretation		Real-life applications	**5.35, 6.36**
Linear equations		Data displays construction		Independence/range/fluency	5.36, 6.37
Quadratic equations		Central tendency			
		Probabilities			
		Contextual probability			

Materials
ECPC Strategy Sheet
Discount Evaluation Task Handout
Water Evaluation Task Handout
Car Comparison Task Handout
Teacher Answer Sheet
Vocabulary Sheet

Learner Prior Knowledge
Students should know how to estimate prices (in dollars), convert between units, construct **2-variable line graphs**, and calculate percentages.

Instructional Activities
Step 1: Depending on your students' level, you may want to briefly review **unit conversions**, percentages, and estimation.

Step 2: *Explain ECPC strategy*. Ask students if they have heard the colloquial saying, "easy peasy" (or "easy peasy lemon squeezy") to mean that something is not difficult. This can be used as a mnemonic device to remember the Estimate-Convert-Plot-Compare strategy that they will be learning. The strategy can be used in almost any situation where the student needs to compare two or more things. It is particularly helpful for the complicated comparisons we make every day. In fact, mastering the ECPC strategy in everyday life can help save hundreds or even thousands of dollars each year (of course, it needs to be combined with an awareness of discount options that are available). The other advantage is mathematical; the ECPC strategy engages students in a **high cognitive level** processing known as "procedures with connections" (Stein & Smith, 1998), which has been shown to increase

problem-solving abilities significantly (Boston & Smith, 2009). Go through the *ECPC Strategy* sheet with students. If the students seemed comfortable with the review in Step 1, cover the strategy quickly.

Step 3: (I do) *Teacher models ECPC strategy through the Discount Evaluation Task*. Pass out the *Discount Evaluation Task* handout to each student. Using the **thinkaloud technique**, read the problem out loud and then give the students time to read it through one more time for understanding. Then comes the most important part of the entire lesson: You must think out loud as you try to "figure out" how to solve the problem. It is critical that the students see all the little twist and turns that go on in your head – this is one of the fundamental components of explicit instruction. Pretend that you have never seen the problem before and say things like, "Hmmm… well, I notice that there are lots of prices, so maybe I should just calculate the answer for each option. No, wait. I have to decide between several options and I don't know exactly how many rolls I am supposed to be. This seems complicated. Since I'm comparing things, maybe I could try the ECPC strategy. So the E stands for estimate; how am I going to estimate these…" Many of the steps within this first task are not particularly challenging, but the combination of using them all to buy 2–5 rolls of toilet paper makes this a Level 5 or 6 task.

Step 4: (we do) *Teacher and class use the ECPC strategy together to work through the Car Comparison Task*. Call on a student to read the task out loud, and then have another student read it out loud a second time. Ask students if this seems like an appropriate time to use the ECPC strategy and press them to explain why. Call on students frequently at each step of the process, prompting them when necessary. However, if a student ever does or says something incorrectly, make sure that you correct it immediately. Try to give the students as much ownership as possible in the solution process, but remember that the goal of Step 4 is to do the problem together.

Step 5: (you do) *Students independently use the ECPC strategy to work through the Water Evaluation Task*. Depending on your class dynamics, either partner students together or have them work individually. Before you pass out the task, explain that you want the students to tackle this problem as independently as possible. After passing out the handouts, walk around the room silently monitoring the students' progress. When you see them run into difficulties, try not to answer their questions directly; instead, remind them of similar situations from the first two tasks.

Step 6: Have each student (or pair) share both the process they used and their final comparisons. Encourage students to discuss the pros and cons of alternative approaches taken.

Step 7: *Making it relevant.* Have students brainstorm examples from their lives of when ECPC strategy might be useful.

Assessment/Evidence (*based on outcome)*

Steps 5 and 6 will serve as evidence of student mastery. During Step 5, the teacher should actively listen to partner discussions for signs of understanding or of misconceptions. If students are working alone, the teacher should have students think out loud as they solve the problem (similar to how the teacher modeled the ECPC strategy out loud in Step 3). During Step 6, allow students the opportunity to modify their solutions based on what they learn from watching others present their solutions.

Exit Slip: You are trying to decide between three brands of cereal. Reechios is on sale for $2.79 a box. Flosted Frakes cost $3.69 a box, but have a "buy 2, get 1 free sale." Organic Carrot Flakes are $5.89 a box, but are 50% off. All the boxes are the same weight. Which cereal is the best deal if you want to buy 2 or 3 boxes? Explain your reasoning.

Teacher Reflection/Lesson Evaluation

Not yet completed.

Next Steps

Have students come up with a specific price comparison they are facing in their lives. This can be as small-scale as choosing which type of bread or milk to buy, or as large-scale as leasing or purchasing a place to live. Ask them to collect actual money data for these decisions and then work through the ECPC process.

Technology Integration

Students can compare discounts for places they actually shop at the following site:
http://www.retailmenot.com

These two articles provide information on comparing lease vs. purchase of a car (much more in-depth than the lesson went).

http://www.ehow.com/about_7330025_car-lease-vs_-buying-cheaper_.html
http://www.insure.com/car-insurance/vehicle-lease-information.html

Purposeful/Transparent

Spending money wisely is an important concern for students. This lesson asks students to use the ECPC strategy in three situations (comparing discounts, comparing modes of transportation, comparing purified water sources).

Contextual

Real life price comparison contexts are used throughout the lesson. Although they are simplified for the purpose of the course, many of the complex elements inherent in price comparisons still remain. Moreover, the comparative discount website provided is an actual site that is excellent for saving money, and most of the prices used for the tasks are actual prices.

Building Expertise

Students synthesize their knowledge of simple percentage calculations and unit conversions in complex multi-step tasks. The final step of plotting the graphs forges connections between algebraic and graphical representations.

References

Boston, M. D., & Smith, M. S. (2009). Transforming secondary mathematics teaching: Increasing the cognitive demands of instructional tasks used in teachers' classrooms. *Journal for Research in Mathematics Education, 40,* 119–156.

Stein, M. K., & Smith, M. S. (1998). Mathematical tasks as a framework for reflection: From research to practice. *Mathematics Teaching in the Middle School, 3*(4), 268–275.

ECPC Strategy

Estimate. Both in real life situations and on standardized tests, we often do not have time to make every calculation precisely, especially in a multi-step process with messy numbers. At its most basic level, estimation is about balancing accuracy with ease. If the students feel they can multiply, divide, add, and subtract the numbers quickly, there is no need to sacrifice accuracy for ease. For example, $3 + $9 + $11 = ?. We could round each number to the nearest $5 and get $25, or the nearest $10 and get $20, but it is just as easy to add the actual numbers and get the exact answer of $23. On the other hand, $15.95 * 4 = ? would be much easier as $16 * 4 = $64 and even simpler (but a bit less accurate) as $15 * 4 = $60. How do you know when you've estimated too much? In a test situation, if your answer comes very close to two of the possible choices, you should probably go back and check your estimation. When you are comparing four or more things to find the best option (like when you go shopping), you can use a rough estimate to determine the best two options and then a good estimate or exact calculation to decide which is the better of the two.

Convert. There are many types of conversions. There are the standard measurement conversions (length, volume, speed) and then there are conversions of discounts. For example, how could you convert "half-off" into something that could be compared with a 30% discount? You could convert the 30% to three-tenths off, or you could convert half-off into 50%. In fact, if you feel more comfortable with decimals, you could convert them both to decimals (50% = .50 and 30% = .30). Although the Convert step comes after the Estimate step, you can always use estimation to make your conversion easier. For example, if you are trying to convert 16 kilograms to pounds and you know that 1 kg = 2.2 lbs, you might estimate that 16 kilograms equals about 32 pounds (16*2 = 32).

Plot. In a simple estimation or conversion, you will probably not need to plot. However, for complicated problems, a nice graph can make your life a lot easier. It can also show you relationships that you never knew existed. Consider a situation at the grocery store where there is a "Buy 3, get 1 free" promotion on Woochies grape juice. Perhaps buying 1 or 2 jugs of the generic brand would be cheaper than 1 or 2 jugs of Woochies, but with the discount, buying 4 jugs of Woochies is better than 4 jugs of the generic brand. In this case, your comparison would depend on how many items you are buying. This can get very complicated, but plotting a graph lets you make comparisons at a glance.

Compare. Once you plot the graph, the hard work is done. In real life, the numbers are just one factor of a comparison. Buying generic grape juice might be cheaper, but perhaps you crave that distinct aftertaste that Woochies leaves in your mouth. The plot allows you to compare different options using numbers, but it is up to you to then factor in everything else you consider important. Of course, in a test situation, you will only be asked to compare the numerical portion.

Rolling in the Money: Teacher Answer Sheet

Estimated answers are approximate and may not match the answers provided below exactly.

Task 1

1. Here's an example of a table with the total price and price per roll for each of the five offers. It would also be fine to create another method of displaying the information (such as a line graph).

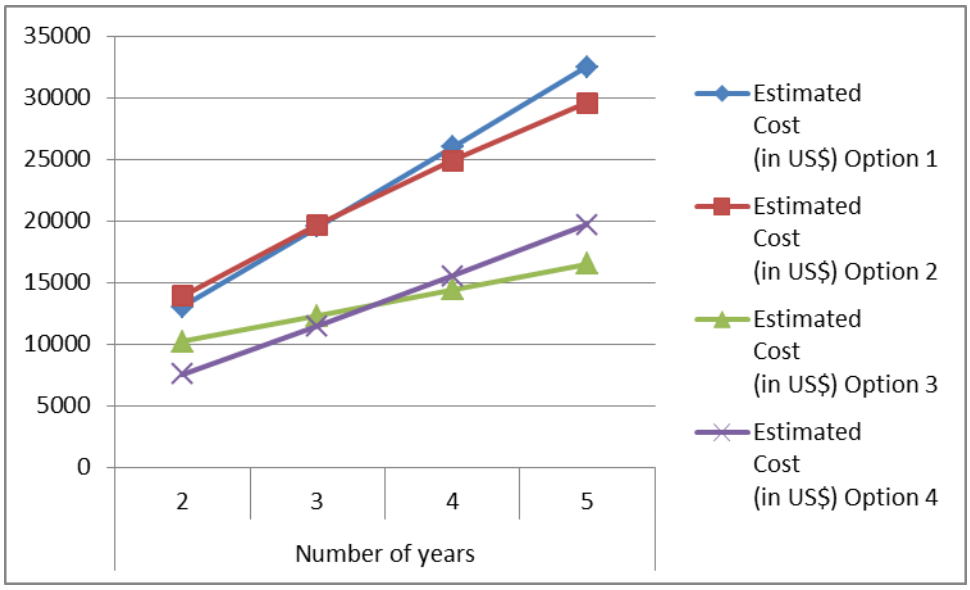
		Number of value packs			
		2	**3**	**4**	**5**
Estimated Price (in US$)	**Offer 1**	24.00	34.50	35.50	46.00
		12.00	11.50	8.88	9.20
	Offer 2	24.00	28.50	38.00	47.50
		12.00	9.50	9.50	9.50
	Offer 3	21.00	30.00	39.00	48.00
		10.50	10.00	9.75	9.60
	Offer 4	21.00	30.00	39.00	48.50
		10.50	10.00	9.75	9.70
	Offer 5	19.00	29.50	35.00	45.50
		9.50	9.83	8.75	9.10

2. Answers may vary with student, but should be supported with logic. Gray boxes in the above chart show some of the lowest prices per value pack.

Task 2

1. Here's an example of a table for each of the four options. In this case, a graph might be more helpful, because it allows students to see trends in the data, as well as connections between algebraic and graphical representation (for options 1, 3, and 4, it is easy to see the linear trend; in option 2, there is an exponential trend).

		Number of years			
		2	**3**	**4**	**5**
Estimated Cost (in US$)	**Option 1**	13000	19500	26000	32500
	Option 2	13,901	19,682	24,868	29,567
	Option 3	10200	12300	14400	16500
	Option 4	7520	11460	15520	19700

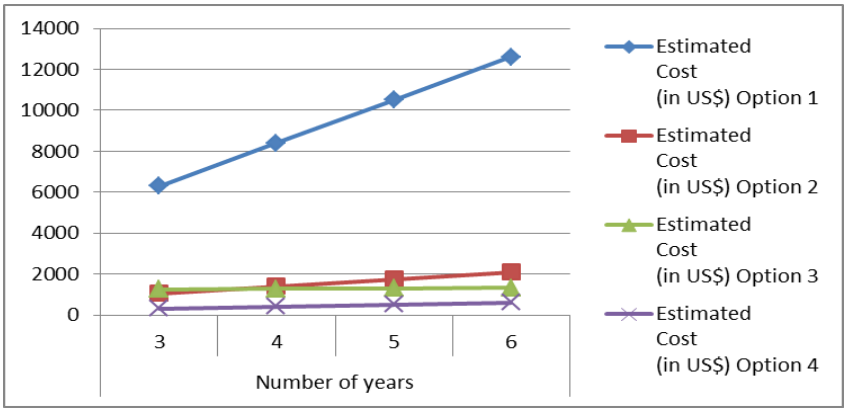


2. Answers may vary with student, but should be supported with logic. Gray boxes in the above chart show some of the lowest prices per value pack.

Task 3

1. Here's an example of a table for each of the four options. In this case, a graph might be more helpful, because it allows students to see trends in the data, as well as connections between algebraic and graphical representation (for options 1, 3, and 4, it is easy to see the linear trend; in option 2, there is an exponential trend).

		Number of years			
		3	**4**	**5**	**6**
Estimated Cost (in US$)	**Option 1**	6300	8400	10500	12600
	Option 2	1050	1400	1750	2100
	Option 3	1240	1270	1300	1330
	Option 4	315	415	515	615



2. In this case, there is a clear answer; Option 4 is the cheapest. However, 2, 3, and 4 are all reasonably similar, and so students may support their decision with other reasons (besides cost).

Rolling in the Money: Vocabulary Sheet

2-variable line graphs – a line graph that plots the dependent variable as a function of the independent variable on the X-Y plane.

High cognitive level – mathematical processing where students make connections between different representations (e.g., graphs and algebra), or where they engage in true understanding of a problem rather than just following a routine.

Thinkaloud technique – a teaching practice where the instructor (or student) attempts to verbalize all of his or her thoughts in solving a problem.

Unit conversions – translating a value from one measurement unit into another (such as meters to feet or meters to centimeters).